Scalar Field EFTs in Cosmology

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Abstract

We review a general effective field theory (EFT) that maps to K-essence and other dark energy theories. We find the mapping of K-essence to the general EFT, and take the quintessence limit of the resultant Lagrangian. We then choose a ϕ^4 potential so that we can review the Weinberg-Coleman potential, vacuum decay and discuss it in light of the preceding.

1 Introduction

One of the most pressing issues about the standard model of particle physics today is its inability to account for some of the observations made at cosmological scales. According to them, granted the standard cosmological model, the universe's energy budget comes largely from a dark sector, so far eluding anything but a phenomenological description: dark matter and dark energy. The former has seen a successful description in terms of a cold, collisionless fluid, though with some remaining observational tensions, whereas the latter typically has been explained as a cosmological constant entering at the level of the Lagrangian, providing an accelerated expansion of space at late times. Although this constant is an allowed term of the symmetries of general relativity, there is some difficulty in explaining its observationally inferred value as a quantum field theoretical evaluation of the vacuum energy contribution from the matter fields would have it 120 orders of magnitude larger (the fine-tuning problem). It is also puzzling why we find ourselves at a time where this term very recently (cosmologically speaking) has become dominant on the expansion of the universe (the coincidence problem). The lack of a theoretically compelling explanation for the manner of the late-time acceleration, our hitherto only motivation for it coming from observations on large scales and finally some recent tensions with observational data under the cosmological constant framework, has led some to consider actions where this effect arise dynamically from matter fields or from extra gravitational degrees of freedom instead.

One example of a class of theories attempting to account for the effect of dark energy by adding a scalar degree of freedom is K-essence [1], building on the more explored quintessence field which uses a similar mechanism to the inflaton field in a slow-roll potential for achieving the negative pressure equation of state for the resultant cosmological fluid. K-essence achieves the dark energy behaviour through the insertion of a non-trivial kinetic term for the scalar degree of freedom and shows some promise in accounting for both the fine-tuning and coincidence problem, although the latter has been claimed to be on the expense of causality [2, 4]. K-essence extends the Einstein-Hilbert action of general relativity to

$$S = \int d^4x\sqrt{-g}\left\{g(\phi)M_{pl}^2R/2 + P(\phi, X)\right\} + S_m(g_{\mu\nu}), \qquad (1)$$

where[1] g is some general function of ϕ, P some function of $X = g_{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ that makes up the non-linear kinetic terms and S_m is the standard matter action, coupled to only the metric.

1.1 Effective Field Theories

In general, an effective action can be expressed intuitively in the path integral formalism as

$$e^{i\Gamma_{EFT}} = \int\prod_j \mathcal{D}\phi_j e^{iS_{UV}(\{\phi_i\})}, \qquad (2)$$

where the j index can span a subset or all of the ϕ_i fields, and ϕ are meant to represent all scalar, spinor, vector and tensor fields. We say that we are 'integrating out' the ϕ_j degrees of freedom, and in the event where j spans all the fields, we are left with the 1-part-irreducible (1PI) effective action, which would include all quantum loop-corrections, even though we only evaluate it at tree-level.

The effective action would in general contain an infinite series of non-renormalisable operators whose (Wilson) coefficients would contain information about the integrated-out degrees of freedom and would have

[1] It can be shown in starting out in the Einstein frame that g is unity in K-essence models, even in the Jordan frame. The two frames should be equivalent up to a conformal transformation of the metric.

to be fixed by observations in a bottom-up construction (explained below). Yet, when there is a clear scale hierarchy between the integrated out (heavy) and remaining (light) degrees of freedom, a dimensional analysis (power counting) will show a suppression of higher dimensional operators and only give a finite amount of non-renormalisable operators to fix to observations at a given precision, and thus a predictive, and often very useful, effective theory. One would typically then simply neglect all on-shell excitations of the heavy particles as the energy available in the kinematics one is considering is not sufficient to excite them. The only trace left of the heavy degrees of freedom are then parametrising their off-shell excitations in internal graphs for external light states.

In the following, what is relevant is a slight modification of the above picture, known as the background field method. Instead of integrating out a heavy particle, one splits ones extra degree of freedom into a dynamical part $\delta\phi$ and a classical background field ϕ_b, so that $\phi = \phi_b + \delta\phi$. One thinks of the non-dynamical field as an external classical field that does not contribute to internal loops (i.e. only exists on-shell). Instead of integrating over it in the path-integral, one inserts its tree-level expectation value. $\delta\phi$ is the quantum fluctuations about the background – it only exists off-shell and will be what one integrates over. The resultant effective action is 1P1 since one has integrated out all the fluctuations, yet one still has external classical states that are evaluated at tree-level.

At first we will find, as in the literature presented here, a classical EFT where we split the scalar into its spatial average $\bar{\phi}(t)$ and deviations from this value $\phi - \bar{\phi} \equiv \pi$. This picture is different since π now has a classical part and exists in on-shell external states. We will neither find the 1P1 action, where the fluctuating part of this field is exactly accounted for in tree-level diagrams, as we wish to be agnostic about the UV-completion of our low-energy theory. Rather, we will approximate it by constructing a general parametrisation of actions up to some order in the smallness parameter $\epsilon \sim \pi$. This means that deviations from the spatial mean is taken to be small. In the literature, π is taken to follow the tree-level equations of motion and is therefore treated classically, which probably is a good approximation at cosmological scales. Still, as an exercise, we will consider it a quantum field in a λ^4 potential and consider corrections at the end. K-essence/quintessence has been applied to quantum cosmology [5, 6] which motivates this consideration. Some K-essence actions can find their UV-completion in string theory, like Dirac-Born-Infeld-like velocity potentials [6].

2 Deriving the EFT

We wish to parametrise the low-energy K-essence class of theories by adding to the Einstein-Hilbert Lagrangian all possible terms consistent with the manifest symmetries of the class of theories up to some order $\mathcal{O}(\epsilon) \sim \mathcal{O}(\pi)$ (bottom-up construction). This is a rather extensive task, so we will review the method without going through the entire derivation. A clever trick to get rid of some redundant terms is to first spontaneously break the time-diffeomorphism invariance of GRs full diffeomorphism invariance, add all possible terms consistent with the remaining manifest symmetries, and then 'unbreak' it using the Stückelberg trick [7, 8, 4]. This utilises the fact that our apparent background cosmological GR solution, the FLRW metric, does not have manifest time-translation invariance. The proceding follows a close analogy to the spontaneous symmetry breaking in the Higgs mechanism, which we will come back to later.

We consider that we have some general scalar field that is monotonic in time, allowing us to choose our time-coordinate so that $\phi(x^\mu, t)$ inverts to $t(x^\mu, \phi)$ and we make the gauge choice so that $\pi = 0$ for each spatial hypersurface (the ADM 3+1 formalism [10] is an intuitive choice here). This is called the unitary gauge in analogy to the Higgs mechanism. $\phi - \bar{\phi} \equiv \pi$ is our Goldstone boson, and we are spontaneously breaking the time translation symmetry. The background field $\bar{\phi}$ is simply a function of time, and the scalar degree of freedom is 'eaten' by the metric. We can then easily extend the action in unitary gauge by including higher order terms in the metric, constrained by the manifest symmetries of K-essence, which we identify here to be spatial diffeomorphism invariance and parity. We will attenuate the terms by assuming that all the overdensities are small on our scales, on order of a weak-potential smallness parameter[2] $\epsilon^2 \sim (\delta g_{tt})^2 \sim \delta g_{ti}^{3/2} \sim \delta g_{ij}$.

Looking at tree-level equations of motion, one finds that spatial derivatives are $\mathcal{O}(\epsilon^{-1/2})$, and so, with 2nd order equations of motion, one needs to include ϵ^2 terms in the Lagrangian to be consistently at $\mathcal{O}(\epsilon)$ [7, 12]. Then, adding all possible terms in the unitary gauge, up to 2nd order (expanded around an FLRW background), one finds, using $- + ++$ convention,

$$\mathcal{L} = M_{pl}^2 f(t) R/2 - \Lambda(t) + c(t) g^{00} + M_2^4(t)(\delta g^{00})^2/2. \tag{3}$$

There are more terms listed in [8], but we find them to be zero for K-essence, and so leave them out for brevity.

[2] We need to restrict ourselves to gauges where this remains satisfied. It is assumed here to be so for there unitary gauge. The scalings are for non-relativistc matter, and are found using the spacetime interval and the virial theorem.

After this one undoes the unitary gauge using the Stückelberg trick that is a transformation $t \to \bar{t}(t) = t + \pi$, and gets the result of [7]. We first need to find how g^{00} transforms, and see

$$g^{00} \to \frac{\partial \bar{t}}{\partial x^\mu}\frac{\partial \bar{t}}{\partial x^\nu} g^{\mu\nu}, \tag{4}$$

which is easily solvable, and leads us to

$$\begin{aligned}\mathcal{L} = &\frac{M_{pl}^2}{2} R - \Lambda(\bar{t}) \\ &- c(\bar{t})\{ g^{00} + 2g^{0\mu}\partial_\mu \pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi\} \\ &+ \frac{M_2^4(t)}{2}\left(\delta g^{00} + 2g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi\right)^2,\end{aligned} \tag{5}$$

where dot is with respect to cosmic time and a is the background scale factor. Note that we have counted spatial derivatives $\mathcal{O}(\epsilon^{-1/2})$. The EFT functions f, Λ, c being functions of $\bar{t}$, they will have to be Taylor expanded in the scalar later. At this point one can find the EFT functions explicitly in terms of our K-essence model above using the mapping equations of [8] or a top-down matching procedure. One should then find the EFT functions of [9] (for $\phi = t$)

$$f(t) = g(\bar{\phi}(t)) = 1, \tag{6}$$

$$c(t) = \dot{\bar{\phi}}^2 P', \tag{7}$$

$$\Lambda(t) = c - \bar{P}, \tag{8}$$

$$M_2^4(t) = \dot{\bar{\phi}}^4 P'' \tag{9}$$

where P' means differentiated with respect to X, which is the only dependency of P on the ADM variables of [8]. We evaluate the rest of the EFT functions to be zero, as expected.

After using the Stückelberg trick we should supposedly have reclaimed the completely covariant action, up to the order that we extended the action in the unitary gauge, yet looking at equation (5) manifest covariance seems to be broken by the EFT-functions, the metric and the derivatives. This is because of our splitting of π and ϕ, and also that we have limited ourselves to gauges where the perturbations of the metric remain small and the relative order of spatial and temporal derivatives is maintained. We assume that within this residual gauge, we can set the $\mathcal{O}(\epsilon^2)$ conformal Newtonian gauge,

$$\begin{aligned}ds^2 = &-(1 + 2\Psi + 2\Psi^2)dt^2 \\ &+ a^2\delta_{ij}(1 - 2\Phi + 2\Phi^2)dx^i dx^j,\end{aligned} \tag{10}$$

which is an approximation of the Poisson gauge[3]. We then find the $\mathcal{O}(\epsilon^2)$ Ricci tensor

$$\begin{aligned}R = &6\left(2H^2 + \dot{H}\right)(1 + 2\tilde{\Psi}) - \frac{2}{a^2}\tilde{\Psi}_{,ii} \\ &+ 6\tilde{\Phi}_{,00} - 6H(\tilde{\Psi}_{,0} - 4\tilde{\Phi}_{,0}) - \frac{4}{a^2}\tilde{\Phi}_{,ii},\end{aligned} \tag{11}$$

where we have chosen $\tilde{\Psi} = \Psi + \Psi^2$ and $\tilde{\Phi} = \Phi - \Phi^2$. We finally get

$$\begin{aligned}\mathcal{L} = &\frac{M_{pl}^2}{2} R - \Lambda(t) - \pi\dot{\Lambda}(t) \\ &- \{c(t) + \pi\dot{c}\}\{2\Psi - 1 + 2\dot{\pi} + \delta^{ij}\partial_i\pi\partial_j\pi/a^2\} \\ &- c(t)\left\{2\Psi(\Psi + \dot{\pi}) + 2\Phi\delta^{ij}\partial_i\pi\partial_j\pi/a^2 - \dot{\pi}^2\right\} \\ &+ \frac{M_2^4(t)}{2}\left\{2(\Psi - \dot{\pi}) + \delta^{ij}\partial_i\pi\partial_j\pi/a^2\right\}^2,\end{aligned} \tag{12}$$

which is our effective action.

3 The Coleman-Weinberg Potential

We will now consider the more explored subcategory of K-essence, 'quintessence'. During inflation, quantum dynamics are expected to become influential on cosmology, but there are no obvious reasons to consider quantum effects in dynamics occurring at cosmological scales in the late-time universe. However in some cases one can constrain parameter spaces by the study of instabilities under quantum corrections. Apart from this false vacuums and domain walls have been studied in the literature. The Higgs field is the only scalar field that we know of in nature, and there are some interesting things to say about its vacuum expectation value, and whether it is at a global or local minimum. In the latter case it could relax to a lower energy vacuum some time in the future. We will first show that the Higgs potential can be studied as a special case of our K-essence EFT, and then study the Coleman-Weinberg potential, which looks at quantum corrections to the ϕ^4 potential

We note that although all fields involved are scalar, both the gravitational scalars are dimensionless whereas $\pi \sim t$, which is the inverse of what we find canonically. To return π and Φ, Ψ to scalar dimension $\sim M$, we redefine their fields

$$M_{pl} = \sqrt{\frac{\hbar c}{G_N}} \Leftrightarrow G_N = \frac{\hbar c}{M_{pl}^2} \equiv \frac{1}{M_{pl}^2}, \tag{13}$$

$$\pi \to \tilde{G}\pi, \quad (\Phi, \Psi) \to \sqrt{G_N}(\Phi, \Psi). \tag{14}$$

For convenience $G_N \equiv G$ in the following. We pick $\tilde{G} = 1/M_\pi^2$ to represent some scale associated with K-essence.

For a superficial inspection of equation (12), let us consider the quintessence scenario. Then $P'' = 0$,

[3] A more rigorous approach might show that consistency would require that we include vector and tensor perturbations $\mathcal{O}(\epsilon^2)$. They are expected to be irrelevant for ΛCDM. We ignore them for simplicity. They are included in [7].

$M_2^4 = \dot{c} = 0$ and $c = \tilde{G}^{-2}$, which can be seen by rescaling $\bar{\phi} = t$ before changing to scalar dimension. Furthermore, assuming a universe without anisotropic stress so that $\Psi = \Phi$, the part of the Lagrangian in π is

$$\mathcal{L} \supset \pi\Box\pi - \tilde{G}\,\pi\dot{\Lambda}(t) - 2M_\pi^2(1+\sqrt{G}\Psi)\,\dot{\pi} - 2\sqrt{G}\Psi\,\partial_i\pi\partial^i\pi - V(\pi,t), \tag{15}$$

where $\Box \equiv \eta_{\mu\nu}\partial^\nu\partial^\mu$. This is simply the Klein-Gordon equation to first order in the gravitational potential (remember we counted $\partial_i = \mathcal{O}(\epsilon^{-1/2}), \partial_t \sim \mathcal{O}(1)$). We also get two tadpoles that would have no effect on a 1P1 action, since they cannot participate in any 1P1 diagrams. This reflects the fact that minimally coupled quintessence simply is an additional matter field.

The choice of quintessence has forced us to pick $P(\phi, X) = -V(\phi)$. We will in the following work in Minkowski background. Motivated by the Higgs particle, let us choose the widely studied ϕ^4 potential

$$V(\pi) = m_0^2\pi^2 + \frac{\lambda}{4!}\pi^4. \tag{16}$$

Now, following [14, 15], we can study the interesting prospect of whether the manifest $\mathbb{Z}_2$ symmetry is spontaneously broken, as would be the case in a nonzero vacuum. Because normal π's will be involved, we will refer to the the numeric $\pi_n = 3.14\ldots$ Using the proper background field method, unlike what we did in the beginning, we separate $\pi = \pi_b + \xi$, where π_b is a classical external field, while ξ contains all quantum fluctuations. We then find

$$V(\pi_b+\xi) = m_0^2(\pi_b^2 + 2\pi_b\xi + \xi^2) + \frac{\lambda}{4!}\left(\pi_b^4 + 4\pi_b^3\xi + 6\pi_b^2\xi^2 + 4\pi_b\xi^3 + \xi^4\right). \tag{17}$$

We can drop the ξ tadpoles. We will also drop the ξ^3, ξ^4 terms, as we assume $\xi \ll \pi$. Then, defining $V_2(\xi) = f(\pi_b)\xi^2$ and $f(\pi_b) = m_0^2 + \lambda\pi_b^2/2$ we have

$$e^{i\Gamma[\pi_b]} = \exp\left(i\int \frac{1}{2}\pi_b\Box\pi_b - V(\pi_b)\right) \times \int \mathcal{D}\xi\, \exp\left[i\int d^4x\left(\frac{1}{2}\xi\Box\xi - V_2(\xi)\right)\right]. \tag{18}$$

The path integrand is a Gaussian. This is helpful, because we can use the formula

$$\int \mathcal{D}\phi\, \exp\left(-\phi A\phi\right) = \sqrt{2\pi_n \det A^{-1}}. \tag{19}$$

Taking the logarithm of both sides of (18) helps us to simplify further (looking only at the path-integral part, disregarding a constant)

$$i\Delta\Gamma[\phi_b] \propto -\frac{1}{2}\mathrm{tr}\ln\left(-\Box + f(\pi_b)\right), \tag{20}$$

where we have used the relation $\ln\det A^{-1} = -\mathrm{tr}(\ln A)$. The states $|X\rangle$, $X \in \{x, p\}$ form complete sets of momentum and position states, so we can use their completeness relations $\mathbf{1} = \int d^4X\,|X\rangle\langle X| = \mathbf{1}\int d^4X\,\langle X|X\rangle$, to first express the trace

$$i\Delta\Gamma[\phi_b] \propto -\frac{1}{2}\int d^4x\,\langle x|\ln\left(1 + \frac{f(\pi_b)}{\Box}\right)|x\rangle\,, \tag{21}$$

where one has pulled out a term $\ln\Box$ that is independent of ϕ_b. At this point [14] assumes $f(\pi_b) = m_{\text{eff}}^2 = $ const since this otherwise would make a difficult calculation. We are bound to do the same, but will motivate it assuming that the scales that ξ and π_b evolve on are widely separated. Then we can insert the completeness relation for momentum states anywhere we want within the braket, and not have to worry about their operation on $f(\phi_b)$, so that

$$i\Delta\Gamma_b \propto -\frac{1}{2}\int d^4x \int \frac{d^4p}{(2\pi^4)}\ln\left(1 - \frac{m_{\text{eff}}^2}{p^2 + i\epsilon}\right), \tag{22}$$

where we define $\partial_\pi^2 V = m_{\text{eff}}^2 = m^2 = \Delta$ for convenience. Looking at the superficial degree of divergence of the momentum integral, we can immediately tell that it is UV-divergent. The IR divergence is cancelled by the p^2 from the integral. Although [14] uses a momentum cutoff regulator, we will use dimensional regularisation and compare. The integral can be put into a more familar form with the derivative method ($\epsilon \equiv \frac{4-d}{2}$)

$$I = \int \frac{d^dp}{(2\pi_n)^d}\ln\left(1 - \frac{\Delta}{p^2}\right), \tag{23}$$

$$\frac{d^2I}{d\Delta^2} = \int \frac{d^dp}{(2\pi_n)^d}\frac{1}{(p^2 - \Delta + i\epsilon_2)^2}, \tag{24}$$

$$= \frac{i}{(4\pi_n)^{2-\epsilon}}\frac{1}{\Delta^\epsilon}\Gamma(\epsilon)\,. \tag{25}$$

Taking the limit $\epsilon \to 0$, we find ($A = \frac{i}{(4\pi_n)^2}$)

$$A(1+\epsilon\ln 4\pi_n)(1-\epsilon\ln\Delta)\left(\frac{1}{\epsilon} - \gamma_E\right)(1+\epsilon\ln\mu^2) = A\left(\frac{1}{\epsilon} - \ln\frac{\Delta}{\mu^2}\right) \tag{26}$$

where γ_E is the Euler-Mascheroni constant and μ comes from keeping the the integral at same dimension $d^4p \to \mu^{4-d}d^dp$. We redefined $1/\epsilon \to 1/\epsilon - \gamma_E + \ln 4\pi_n$. Integrating back, we find

$$\Delta\Gamma_b \propto -\frac{VT}{64\pi_n^2}\left(c_1 + c_2 m_{\text{eff}}^2 + m_{\text{eff}}^4\ln\frac{\mu^2}{m_{\text{eff}}^2},\right) \tag{27}$$

with c_i integration constants, both divergent, agreeing with the hard cutoff regularisation of [14]. We find e.g. $c_1 = Am_{\text{eff}}^4/2\epsilon$ simply from the double integral over $1/\epsilon$. In [14] they have the logarithmic divergence the other way around, because the cutoff is taken to ∞, while in dimensional regularisation, μ is arbitrary, but often associated with some scale of the problem.

The divergences should be removable by renormalisation of λ, m, Λ by adding counterterms that we assume are $\mathcal{O}(\lambda_R^2)$, where the bare $\Lambda_b = 0$ is a constant term of the Lagrangian. Then picking a subtraction scheme, like modified minimal subtraction $\overline{\text{MS}}$ where we choose a finite part of the divergence to exactly cancel the $-\gamma_E + \ln 4\pi_n$ terms, we can find the finite effective potential. Instead we will pick a scheme that is more relevant to our consideration of the potential. First we state

$$V_{\text{eff}} \equiv V(\phi_b) + c_1 + c_2 m^2 + \frac{1}{64\pi_n^2} m^4 \ln\left(\frac{\mu^2}{m^2}\right). \quad (28)$$

From the renormalisation, all bare quantities are replaced $X \to Z_X X_R = (1+\delta_X)X_R$. The question of whether $\mathbb{Z}_2$ is a spontaneously broken symmetry (SSB) is somewhat imprecise. We want to know whether the physical degree of freedom π_b undergoes SSB owing to quantum corrections, but need to expand it around its minimum, which is why we used the background field method. We define the classical mass $m = V''(0)$. We then define our renormalisation conditions $V''_{\text{eff}}|_{\pi_b=0} = m_R^2 = 0 = \Lambda_R = V_{\text{eff}}(0)$. We will not have use for renormalisation of the field strength here, but it would typically be defined $Z_\pi(0) = 1$, but could not be so here; It is afflicted by the same problem of an IR divergence that we will have when we try to set the standard condition for the coupling $\lambda_R = V^{(4)}(0)$, that

$$\lim_{m\to 0} V^{(4)} \sim \lim_{m\to 0} \ln\frac{\mu^2}{m^2} \to \infty. \quad (29)$$

We need instead to choose some arbitrary scale where we define our renormalisation condition, where it is fixed from an observation for example.

Writing explicitly the counterterms

$$V_{\text{eff}}(\pi) \supset \Lambda_R Z_\Lambda + \frac{1}{2} m_R^2 \pi^2 Z_m + \frac{\lambda_R}{4!}\pi^4 Z_\lambda \quad (30)$$

Let's write $V_{\text{eff}}^{(4)}(\pi_R) = \lambda_R$ for some scale π_R. For example for δ_m, δ_Λ, we find

$$V_{\text{eff}}^{(2)}(0) = c_2\lambda + \frac{m_0^2\lambda}{64\pi_n^2}\left(2\ln\left(\frac{\mu^2}{m_0^2}\right) - 1\right) \quad (31)$$

$$\equiv -\delta_m m_R^2 \quad (32)$$

$$V_{\text{eff}}(0) = c_1 + \frac{1}{64\pi_n^2} m_0^4 \ln\left(\frac{m_0^2}{\mu^2}\right) \equiv -\delta_\Lambda \Lambda_R. \quad (33)$$

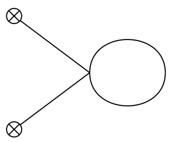
Doing similarly for our other conditions we find that dependence on μ cancels, as it should, and that

$$V_{\text{eff}}(\pi_b) = \frac{1}{4}\pi_b^4\left\{\lambda_R + \frac{3\lambda_R}{32\pi^2}\left[\ln\left(\frac{\phi^2}{\phi_R^2}\right) - 25/6\right]\right\}. \quad (34)$$

Due to our renormalisation conditions this should evaluate to a zero mass for the background field. If it will continue to do so depends on whether the potential is at a minimum. Similarly, we can conclude about whether $\langle \pi_b \rangle$, which should be at a minimum, is 0. What we see is that although $V_{\text{eff}} \to 0$ as $\phi \to 0$, because of the divergent logarithm in the bracket, V_{eff} will change sign in the neighbourhood of 0 due to the quantum correction, and gives therefore a maximum instead. Instead, a minimum can be found at

$$\lambda_R \ln\frac{\langle\phi\rangle}{\phi_R^2} = \frac{11}{3}\lambda_R - \frac{32}{3}\pi^2 \simeq 105. \quad (35)$$

This was an interesting observation, but we see that our perturbation analysis has broken down due to the order of the logarithmic term, and we therefore need to make a resummation of divergent loop integrals. This is what Weinberg and Coleman did in their approach [15]. At 1-loop $\mathcal{O}(\xi^2)$ order one can effectively represent all interactions between the external background field and internal fluctuating field as a series of diagrams with terms of n external pair interactions. For n=1:



$$= \int \frac{d^4p}{(2\pi)^4}\frac{1}{2n}\left(\frac{\frac{1}{2}\lambda\pi_b^2}{k^2+i\epsilon}\right)^n\Bigg|_{n=1}. \quad (36)$$

The 1/2 factor is a symmetry factor for the vertex, we can recognise the numerator from (17). The $1/2n$ factor is a symmetry factor of the whole diagram, counting discrete symmetries of rotation and reflection. We therefore exchange (21) with

$$i\Delta\Gamma = iVT\int\frac{d^4p}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{\frac{1}{2}\lambda\pi_b^2}{k^2+i\epsilon}\right)^n, \quad (37)$$

which we recognise as a logarithm series for $-\ln(1-x)$ for small x, so that we can resum

$$= -\frac{1}{2}VT\int\frac{d^4p}{(2\pi)^4}\ln\left(1 - \frac{\lambda\pi_b^2/2}{p^2+i\epsilon}\right), \quad (38)$$

which is looking familiar, though slightly different, now $m_{\text{eff}}^2 \to \lambda\pi_b^2/2$. Looking at (24), it is clear that π_b plays the role as an IR cutoff, such as in Pauli-Villars regularisation.

The same renormalisation scheme then gives

$$\delta_m = -c_2\lambda/m_R^2, \quad \delta_\Lambda = -c_1/\Lambda_R \quad (39)$$

$$\delta_\lambda = \frac{\lambda_R}{64\pi^2}\left[25 - 3\ln\left(\frac{2\mu^2}{\lambda\pi_R}\right)\right], \quad (40)$$

which inserted into the effective potential (28) (with $m^2 \to \lambda\pi_b^2/2$) gives the same type of behaviour as our previous equation, meaning we need next-order loops. A way to resum higher orders is to use the RGE equations. They let us use the fact that predictions should be independent of the arbitrary scales where the theory is defined. One can find the potential in terms of the 4-point scattering amplitude, like

$$V_{\text{eff}}(\pi_b) = -\frac{\pi_b^4}{4!}\mathcal{M}_4(\pi_b, \pi_R). \tag{41}$$

Since the S matrix is an observable, $\mathcal{M}_4$ should be independent of π_R up to a phase. We can then find the Callan-Symanzik equation for the 4-point function, renaming $\pi_R = \mu$,

$$\mu\frac{d}{d\mu}\mathcal{M}_4 = \left(\mu\partial_\mu + \beta\frac{\partial}{\partial\lambda_R} + \gamma_m\frac{\partial}{\partial m_R} + 2\gamma\right)\mathcal{M}_4 = 0, \tag{42}$$

with $\beta = \mu d\lambda_R/d\mu$ the beta-function of the running coupling, $\gamma_m = (\mu/m_R)dm_R/d\mu$ the anomalous mass dimension and $\gamma = (\mu/Z_\pi)dZ_\pi/d\mu$ is the anomalous dimension of π. We put this solution beyond the scope of this review. A more comprehensive account of the metastability of the Higgs vacuum including the entire standard model in 2-loop order and resummation to 3-loop, performed in [17], finds that a stable vacuum requires $m_h > (129.4 \pm 1.8)$GeV, which means that the observed Higgs at 125GeV indicates an unstable vacuum, granted the standard model, to a significance of 98% according to [17].

4 Discussion and Conclusion

The topics spanned in this review are loosely connected around the topics of effective field theories (both classical and quantum) and scalar fields and their symmetry breaking. An outline of an interesting frontier has been sketched, where both classical and quantum dynamics maintain relevance at cosmological scales. Starting out with a general scalar degree of freedom cosmological EFT, we specialised to K-essence, an interesting candidate for inflation and dark energy using non-linear kinetic terms instead of a slow-roll potential, which we unfortunately did not find time to study in detail. We then specialised to quintessence that does use a slow-roll potential for its negative pressure behaviour. A special case of quintessence could be the Higgs particle, although without beyond-standard model particle physics [17] finds that it is not favoured by the data. Still, the intriguing possibility to have some unknown effect to allow the Higgs to develop a negative pressure and drive the expansion of spacetime instead of some entirely new undetected degree(s) of freedom deserves a closer scrutiny. For example, [18] argues that a non-minimal coupling with the Ricci scalar $\xi|H|^2R$ could account for inflation by flattening the ϕ^4 potential at Planck scales, giving conditions for slow roll. However [17] notes loss of perturbative unitarity at some scale $M_{pl}/\sqrt{\xi}$, which must (but can [19]) be saved at the cost of minimality, and the most favoured Higgs mass allowing for inflation at sub-breakdown scales is disfavoured by the mass of the top-quark at 2σ. The possibility of inflation happening at the transition from a false vacuum to the current has also been investigated [17], but minimality is also here so far lost due to the difficulty in exiting inflation. It would anyway be interesting if a scalar particle and especially the Higgs through some dynamic, maybe yet undiscovered mechanism specific to cosmological or early universe environments could develop novel phenomenology (e.g. non-linear kinetic terms) and through them a natural causation for dark energy/inflation. Such a bold statement should however be met with a thorough analysis, consideration of observational constraints and a broad coverage of the literature on the subject of which only a tiny fraction has been covered here.

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