Project for FYS5190/FYS9190 Supersymmetry
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An EFT for Neutralinos and Charginos

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We consider MSSM with RPC, limiting ourselves to describing a subset of the interactions of neutralinos and charginos, without quarks and only one generation of fermions. After a short introduction to EFTs, we look at a specific diagram and use it as an example as we construct a formalism for finding effective operators parametrising the effects of heavy internal propagators, and which we can add to the Lagrangian with the heavy field put to zero. We compare to the MSSM treatment.

1 Field Content

In SUSY, we generalise the Lagrangian density to a density in superspace so that

$$S = \int d^4x \int d^4\theta \mathcal{L}, \quad \mathcal{L} = \mathcal{L}_{\text{kin}} - W,$$

where $\mathcal{L}_{\text{kin}}$ is the kinetic terms while W is the superpotential. Even in MSSM, these terms are pretty extensive, and for simplicity, we'll focus on interactions amongst the electroweak gauginos and higgsinos, excluding much of the field content.

The reason why this is possible is through the combined restrictions in interactions caused by demanding colour- and R-parity conservation (RPC), where

$$R = (-1)^{2s+3B+L},$$
$$s, B, L \sim \text{spin, baryon number, lepton number},$$

and all SM particles have $R = 1$ while all superpartners have $R = -1$, [1]. We'll then only be able to produce superpartners in pairs, and a superpartner won't be able to decay into only 'normal' particles. Quarks and gluons, squarks and gluinos, will effectively be detached from these interactions, as you can't have colour charge production (You could have several coloured particles produced whose net colour is zero, but at low energies the production amplitudes would be suppressed).

We'll also assume a mass hierarchy so that, at low energies, there can never be on-shell production of heavy- from light degrees of freedom. This will further restrict our considerations and motivate the EFT treatment.

1.1 Neutralinos & Charginos

Both hadron and lepton colliders, if their energies are high enough, might receive their cleanest signature of SUSY from its extended fermion sector [2], which therefore are of much interest to us. The only supersymmetric fermions are the gauginos and the higgsinos, which are the partners of gauge bosons and higgs scalars respectively.

Especially interesting are the superpartners of the electroweak sector gauge bosons – $\tilde{B}^0, \tilde{W}^0$ and $\tilde{W}^{\pm}$. Through EWSB, they mix with the higgsinos to form eigth massive fermions, stated in their mass eigenstates as:

$$\tilde{\chi}_i^0 = N_{i1}\tilde{B}^0 + N_{i2}\tilde{W}^0 + N_{i3}\tilde{H}_d^0 + N_{i4}\tilde{H}_u^0,$$
$$\tilde{\psi}_j^{\pm} = n_{j1}^{+}\tilde{W}^{+} + n_{j2}^{+}\tilde{H}_u^{+} + n_{j3}^{-}\tilde{W}^{-} + n_{j4}^{-}\tilde{H}_d^{-},$$

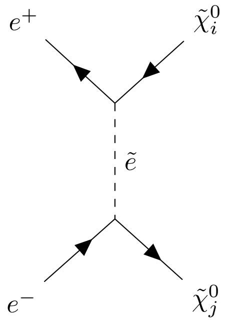
which for $i \in 1,2,3,4$ and $j \in 1,2$ give the neutralinos and charginos respectively. Conventionally, these are arranged so that the first element is the lightest one, and then increasing mass with index.

In the limit where EWSB is a small effect, the mass eigenstates would be roughly similar to the weak eigenstates.

1.2 Interactions

We consider some interesting interactions found in Haber & Kane [2] that satisfies our restrictions.

For concreteness and simplicity, we'll assume the lightest neutralino to be the LSP, and there to be some unification scale where the gaugino masses and couplings become unified. The remaining free parameters of the theory translate to uncertainties of production rates and decay branching ratios, so we'll for the purposes of this paper, not knowing the dominating channels of the true theory, pick some specific diagrams, from which our method will be made clear and extendable to other diagrams. Specifically, we find it interesting to examine electron-positron reactions, seen as this might be a starting point in lepton colliders, or a possible subdiagram for hadron colliders. Our chosen diagram is (from left to right):



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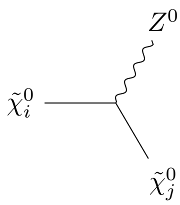
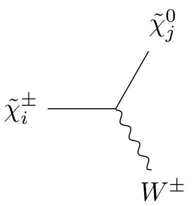
with the following decay channels:





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We could specifically combine these to look at $e^+e^- \to \tilde{\chi}_1^0, \tilde{\chi}_1^0 Z^0, \tilde{\chi}_1^0 W^+W^-$, under the assumption that $m_W, m_Z, m_{\tilde{\chi}_1^0} \ll \min(m_{\tilde{\chi}_2^0}, m_{\tilde{e}})$.

From lack of time/space, we'll even further restrict our focus to only consider direct LSP production from the first diagram.

Of course, again, there are several other possibilities that would contribute to the total cross section for these reactions other than the ones constructed from these subdiagrams.

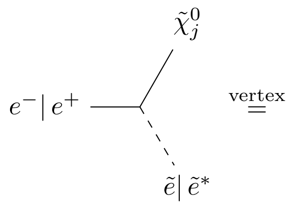
2 MSSM Treatment

From Haber & Kane [2], we find the relevant interaction terms in the Lagrangian (no sum):

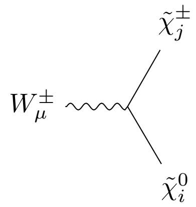
$$\mathcal{L}_{ee\tilde{\chi}_j^0} = -g\bar{e}P_R\tilde{\chi}_j^0\tilde{e}_L\mathcal{T}_j,$$
$$+ g\bar{e}P_L\tilde{\chi}_j^0\tilde{e}_R\mathcal{N} + h.c.,$$
$$\mathcal{L}_{\tilde{\chi}_j^\mp\tilde{\chi}_i^0 W^\pm} = gW_\mu^\pm\bar{\tilde{\chi}}_i^0\gamma^\mu\mathcal{O}_{ij}\tilde{\chi}_j^\mp,$$
$$\mathcal{T}_j = -[N_{j2} + N_{j1}\tan\theta_w]/\sqrt{2},$$
$$\mathcal{N} = -\sqrt{2}\tan\theta_w N_{22}^*, \quad \mathcal{O}_{ij} = O_{ij}^L P_L + O_{ij}^R P_R,$$

where O_{ij} is made up of a combination of the mixing matrices of neutralinos and charginos and is given explicitly in [2]. We'll set $\tilde{e}_L = 0$ and $\tilde{e}_R = \tilde{e}$ for simplicity.

From these we find for Feynman rules in momentum space:



$$\overset{\text{vertex}}{=} -igP_R\mathcal{N}^*|igP_L\mathcal{N}$$



$$\overset{\text{vertex}}{=} ig\gamma_\mu\mathcal{O}_{ij},$$

while the propagators and external leg contractions look as usual.

With these rules, we find for the $e^+e^- \to \tilde{\chi}_1^0$ production (at minimal vertices):

$$\frac{1}{4}\sum_{rstu}|\mathcal{M}|^2 = \frac{1}{4}\left(\frac{g^2|\mathcal{N}|^2}{t - m_{\tilde{e}}^2}\right)^2 \times$$
$$\mathrm{Tr}\left\{P_L(\not{k}_2 + m_{\tilde{\chi}_1^0})P_R(\not{p}_2 + m_e)\right\}\mathrm{Tr}\left\{P_L(\not{k}_1 - m_{\tilde{\chi}_1^0})P_R(\not{p}_1 - m_e)\right\}$$
$$= \frac{1}{4}\left(\frac{g^2|\mathcal{N}|^2}{t - \tilde{m}_{\tilde{e}}^2}\right)^2 (m_e^2 + m_{\tilde{\chi}_1^0}^2 - t)^2,$$

where we denoted fermion momenta p and neutralino momenta k, electron-line 1 and positron line 2. As a prediction for the effective theory amplitude, we may simply set $t = 0$, because the selectron mass is much larger.

3 Effective Field Theory

Imagine that you have some very complicated, UV-complete theory and want to find a cross section for some interaction. In some cases, realising that you're working at an energy scale where one or more of the degrees of freedom effectively freezes out, their propagators being suppressed by their masses squared and energy not being available to produce them off-shell from lighter degrees of freedom, might offer great simplifications. At a lowest level, you might simply realise that a 2-vertex interaction between light particle external states with an internal heavy degree of freedom may be described as a single effective vertex contracting with the same end states.

3.1 The Wilson Action

More rigorously, one can start out from the path integral for an n-point vacuum correlation function, expressed in terms of the generating functional that couples the fields to external currents, in the limit where the currents vanishes. Expressing $\mathcal{D}\phi_i[\mathcal{D}\psi\mathcal{D}\bar{\psi}]_j = \mathcal{D}\Phi_i$ and $|\Omega\rangle \sim$ the true vacuum (the coupling to external currents of the fermion fields requires some extra care from them being anticommuting Grassmann numbers; We'll notationally treat things as scalar here):

$$\langle\Omega|\,\Theta_1\ldots\Theta_N\,|\Omega\rangle = \exp\{-iW[J]\} \times \int \mathcal{D}\Phi_i[\Theta_1\ldots]\exp\left\{i\int d^4x\,[\mathcal{L}+J_i\Phi_i]\right\},$$

where the functional $\exp\{W\}$ is equal to the integral for $N=0$. Then we're able to express the scalar fields as functional derivatives with regards to the external currents, and once we've found the generating functional and the propagator of the free-fields, we only need to know the interaction Lagrangian in order to find observables to any order.

Following Burgess [3] and Weinberg [4], one may realise that $W[J]$ then generates the connected correlations of the operators, while we might reexpress this in terms of generators of connected 1-part-irreducible (1PI) graphs by a Legendre transform to switch variable from the external currents to expectations of the fields under the external currents (were we leave the field index implicit):

$$\exp\{i\Gamma[\Phi_J]\} = \int \mathcal{D}\Phi\exp\left\{i\int d^4x\,[\mathcal{L}(\Phi_J+\Phi)+J\Phi]\right\}.$$

Next, we can realise that we wont have external heavy fields, so we drop their coupled currents from the onset. Finally, we recognise that we can split the path integration into a light energy and heavy energy part and get:

$$\begin{aligned}&\exp\{i\gamma[\Phi_J]\}\\&=\int[\mathcal{D}]_{l.e.}\exp\left\{i\int d^4x[\mathcal{L}_W(\Phi_J+\Phi_{l.e.})+J\Phi_{l.e.}]\right\},\\&\exp\{iS_W[\Phi_J+\Phi_{l.e.}]\}\\&=\int[\mathcal{D}\Phi]_{h.e.}\exp\{iS[\Phi_J+\Phi_{l.e},\Phi_{h.e.}]\}.\end{aligned}$$

This latter expression, solves for an effective action that we call the Wilson action, to be inserted into the generating functional γ. The Wilson action over 1 light particle irreducible (1LPI) graphs can be shown to enter into the expression for Γ the same way as the tradiational action; You can thus treat them as the same, and it is therefore tempting to consider SM to already be some Wilson action of a yet more fundamental, UV-complete unknown theory.

4 EFT Treatment

We assume that the generating functionals are computable in a semiclassical loop expansion so that

$$\gamma = \gamma_t + \gamma_{\text{1-loop}} + \ldots, \quad \mathcal{L}_W = \mathcal{L}_W^t + \mathcal{L}_W^{\text{1-loop}} + \ldots$$

where t-index indicates three level, meaning we evaluate the classical action, traded the operators for their expectations under the external currents.

4.1 Tree Level Effective Action

Starting out from the interaction terms given in section 2 + free field Lagrangian terms, we can set out to find the tree level approximation to the effective action.

First we eliminate the heavy fields, expressing them in terms of the light fields by their classical equations of motion. For a field, the classical equations of motion are

$$\frac{\partial\mathcal{L}_0}{\partial\Phi} - \partial_\nu\frac{\partial\mathcal{L}_0}{\partial(\partial_\nu\Phi)} = \frac{\partial V_I}{\partial\Phi} - \partial_\nu\frac{\partial V_I}{\partial(\partial_\nu\Phi)},$$

where you also get an equation in $\bar{\Phi}$. Giving here for the heavy dofs:

$$\begin{aligned}\Box\tilde{e} &= m^2\tilde{e} - \frac{g\mathcal{N}^*}{2}\bar{\tilde{\chi}}_j^0 P_R e,\\ \Box\tilde{e}^* &= m^2\tilde{e}^* + \frac{g\mathcal{N}}{2}\bar{e}P_L\tilde{\chi}_j^0,\\ i\partial\!\!\!/\tilde{\chi}_j^0 &= m_{\chi_j}\tilde{\chi}_j^0 + gP_R e\tilde{e}^*\mathcal{N}^*,\\ -i\partial_\mu\bar{\tilde{\chi}}_j^0\gamma^\mu &= m_{\chi_j}\bar{\tilde{\chi}}_i^0 + g\bar{e}P_L\bar{e}\mathcal{N}.\end{aligned}$$

This is getting a bit much, so we put $\tilde{\chi}^\pm = \tilde{\chi}_{i>1}^0 = 0$. Inverting the equations of motion for the selectron:

$$\begin{aligned}\tilde{e}(e,\tilde{\chi}) &= -(\Box - M_{\tilde{e}}^2)^{-1}(g\mathcal{N}^*\bar{\tilde{\chi}}_i^0 P_R e)\\ &= -\int d^4y\frac{g\mathcal{N}^*}{2}\bar{\tilde{\chi}}_1^0 P_R e\int\frac{d^4p}{(2\pi)^4}\frac{i}{p^2-M_{\tilde{e}}^2}e^{-ip(x-y)}\\ &\simeq \int d^4y\frac{g\mathcal{N}^*}{2}\bar{\tilde{\chi}}_1^0 P_R e\,\delta^{(4)}(x-y)/\mathcal{M}_{\tilde{e}}^2\\ &= \frac{g\mathcal{N}^*}{2M_{\tilde{e}}^2}\bar{\tilde{\chi}}_1^0 P_R e,\\ \tilde{e}^*(e,\tilde{\chi}) &= \cdots = -\frac{g\mathcal{N}}{2M_{\tilde{e}}^2}\bar{e}P_L\tilde{\chi}_1^0.\end{aligned}$$

We can find next orders by noting (non-trivially) that you may do a Taylor in the free propagator:

$$(\Box - M^2)^{-1} = -\frac{1}{M^2} + \frac{\Box}{M^4} + \ldots,$$

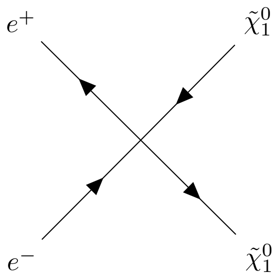
Going back to the action, inserting these equations, we find:

$$\begin{aligned}{}^{(1)}\mathcal{L}_W^{t,I} &= -g|\mathcal{N}\bar{e}P_L\tilde{\chi}_1^0|^2/M_{\tilde{e}}^2,\\ {}^{(2)}\mathcal{L}_W^{t,I} &= -g^2|\mathcal{N}\partial_\mu(\bar{e}P_L\tilde{\chi}_1^0)|^2/M_{\tilde{e}}^4\end{aligned}$$

which is what we were looking for – an effective 4-vertex, hiding away the internal heavy selectron propagator. We also note that in the limit where $M_{\tilde{e}} \to \infty$, this interaction shuts down completely.

4.2 Amplitude

It is then straightforward to find the lowest order amplitude; We see:



$$\overset{\text{vertex}}{=} -ig^2|\mathcal{N}|^2 P_L P_R / M_{\tilde{e}}^2,$$

where the projection operators have to be placed in between the correct external leg contractions, as seen from the effective term (P_L on the positron line).

Finding the amplitude:

$$\frac{1}{4}\sum_{rstu}|\mathcal{M}|^2 = \frac{1}{4}\left(\frac{g|\mathcal{N}|}{M_{\tilde{e}}}\right)^4 \mathrm{Tr}\{P_L(\not{k}_1 - m_\chi)P_R(\not{p}_1 - m_e)\}$$
$$\times\, \mathrm{Tr}\{P_L(\not{k}_2 + m_\chi)P_R(\not{p}_2 + m_e)\},$$

which we already now see agrees with the MSSM calculation above for $t = 0$.

We note that 2 of these 4-vertices put together can give an $e^+e^- \to e^+e^-$ scattering. We can treat then the LSP as heavy compared to the electron and integrate it out, finding an effective term to be added to the SM Lagrangian.

5 Conclusion

We have reviewed a framework for exploiting energy scale hierarchies in Lagrangians by constructing effective Lagrangians from integrating out the heavy degrees of freedom, parametrising their effect in terms of new effective terms.

We showed explicitly how to perform this procedure at lowest order in both couplings and mass for an example reaction.

Several simplifying measures have been made, among others: Only considering lowest level, disregarding diagrams contributing to the same reaction but with other internal propagators that may also be treated as independent heavy or light degrees of freedom, heavy external states and subsequent decay have been disregarded, and only one of the 2 selectrons have been taken into account, although we expect the other term to easily be included in the analysis by similar method; Our main emphasis has been the formalism.

An interesting step forward would be to consider higher order terms (in $1/M^2$), and also 1-loop order, where one can consider the effect of the heavy fields on RG flows of coupling constants and masses of the light fields.

Also it would be nice to see a complete MSSM effective Lagrangian, leaving maybe only the LSP as a light sparticle, integrating out the rest, finding the first correction to SM.

We leave it all as an exercise to the interested reader!

References

[1] P. Batzing, A. Raklev, *Supersymmetry, Lecture notes for FYS5190/FYS9190* (2019).

[2] H. E. Haber, G. L. Kane, Phys. Rept. **117** (1985) 75-263.

[3] C. P. Burgess, *Introduction to Effective Field Theory* (2007)

[4] S. Weinberg, *The Quantum Theory of Fields, Volume 2.* 21st printing (2019) 63-77.